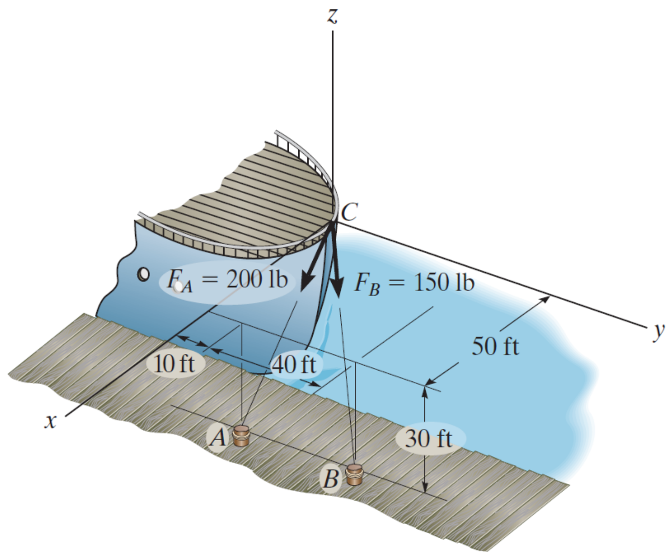
Problem 2-101

The two mooring cables exert forces on the stern of a ship as shown. Represent each force as as Cartesian vector and determine the magnitude and coordinate direction angles of the resultant.



Prob. 2–101

[TYPO: This should be "a."]

Solution

Write the position vectors to the points A, B, and C.

$$\mathbf{r}_A = \langle 50, 10, -30 \rangle \text{ ft}$$

$$\mathbf{r}_B = \langle 50, 50, -30 \rangle \text{ ft}$$

$$\mathbf{r}_C = \langle 0, 0, 0 \rangle \text{ ft}$$

The position vector going from C to A is

$$\begin{aligned} \mathbf{r}_{CA} &= \mathbf{r}_A - \mathbf{r}_C \\ &= \langle 50, 10, -30 \rangle \text{ ft.} \end{aligned}$$

Its magnitude is

$$\begin{aligned} |\mathbf{r}_{CA}| &= \sqrt{(50)^2 + (10)^2 + (-30)^2} \text{ ft} \\ &= 10\sqrt{35} \text{ ft.} \end{aligned}$$

Divide $\mathbf{r}_{CA}$ by its magnitude to get a unit vector in the same direction.

$$\hat{\mathbf{u}}_{CA} = \frac{\mathbf{r}_{CA}}{|\mathbf{r}_{CA}|} = \frac{\langle 50, 10, -30 \rangle}{10\sqrt{35}}$$

The force $\mathbf{F}_A$ can now be written.

$$\mathbf{F}_A = F_A \hat{\mathbf{u}}_{CA} = 200 \, \frac{\langle 50, 10, -30 \rangle}{10\sqrt{35}} \text{ lb} \approx \langle 169, 33.8, -101 \rangle \text{ lb}$$

On the other hand, the position vector going from C to B is

$$\begin{aligned} \mathbf{r}_{CB} &= \mathbf{r}_B - \mathbf{r}_C \\ &= \langle 50, 50, -30 \rangle \text{ ft.} \end{aligned}$$

Its magnitude is

$$\begin{aligned} |\mathbf{r}_{CB}| &= \sqrt{(50)^2 + (50)^2 + (-30)^2} \text{ ft} \\ &= 10\sqrt{59} \text{ ft.} \end{aligned}$$

Divide $\mathbf{r}_{CB}$ by its magnitude to get a unit vector in the same direction.

$$\hat{\mathbf{u}}_{CB} = \frac{\mathbf{r}_{CB}}{|\mathbf{r}_{CB}|} = \frac{\langle 50, 50, -30 \rangle}{10\sqrt{59}}$$

The force $\mathbf{F}_B$ can now be written.

$$\mathbf{F}_B = F_{CB} \hat{\mathbf{u}}_{CB} = 150 \, \frac{\langle 50, 50, -30 \rangle}{10\sqrt{59}} \text{ lb} \approx \langle 97.6, 97.6, -58.6 \rangle \text{ lb}$$

Add the two forces to get the resultant.

$$\begin{aligned} \mathbf{F}_R &= \mathbf{F}_A + \mathbf{F}_B \\ &= 200 \, \frac{\langle 50, 10, -30 \rangle}{10\sqrt{35}} \text{ lb} + 150 \, \frac{\langle 50, 50, -30 \rangle}{10\sqrt{59}} \text{ lb} \\ &\approx \langle 267, 131, -160 \rangle \text{ lb} \end{aligned}$$

Its magnitude is

$$\begin{aligned} |\mathbf{F}_R| &\approx \sqrt{(267)^2 + (131)^2 + (-160)} \text{ lb} \\ &\approx 338 \text{ lb.} \end{aligned}$$

Divide the resultant by its magnitude to get a unit vector in the same direction.

$$\frac{\mathbf{F}_R}{|\mathbf{F}_R|} \approx \frac{\langle 267, 131, -160 \rangle}{338}$$

The direction angles for the resultant can now be found.

$$\begin{cases} \cos\alpha \approx \dfrac{267}{338} \\ \cos\beta \approx \dfrac{131}{338} \\ \cos\gamma \approx -\dfrac{160}{338} \end{cases} \quad \rightarrow \quad \begin{cases} \alpha \approx 37.8^\circ \\ \beta \approx 67.1^\circ \\ \gamma \approx 118^\circ \end{cases}$$